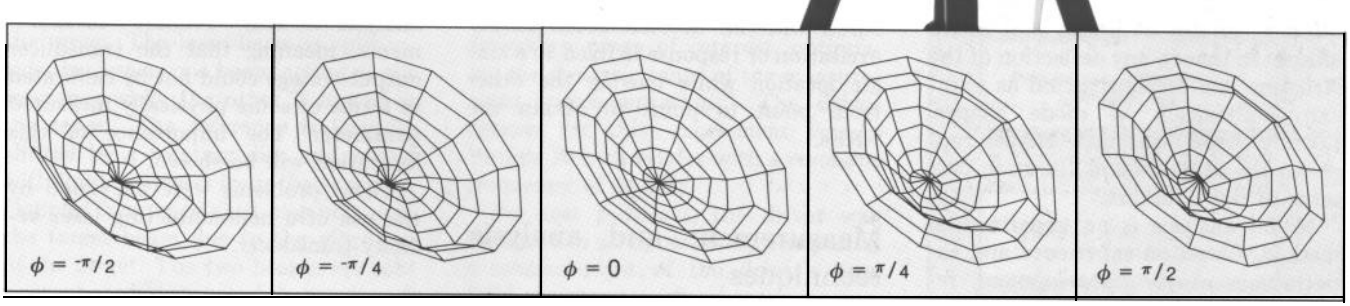
Application Notes

Measuring the Non-rigid Behaviour of a Loudspeaker Diaphragm using Modal Analysis



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Measuring the Non-rigid Behaviour of a Loudspeaker Diaphragm using Modal Analysis

by Christopher J. Struck, Brüel & Kjær

Introduction

When a loudspeaker vibrates at low frequencies, it behaves as a rigid piston, with all parts of the diaphragm moving in phase. Above a certain frequency, however, the diaphragm behaviour will become more complex. To examine this phenomenon, it is necessary to make reliable measurements of the diaphragm motion at a number of different points.

Until recently, accurate vibration measurements on light and delicate structures have been troublesome, if not impossible, to make. The reason for this was that traditional methods required the mounting of a transducer on the test object. This problem can now be solved by using the Brüel & Kjær Laser Velocity-Transducer Set Type 3544, which avoids the mechanical contact with the test object, and together with a Brüel & Kjær Dual Channel Signal Analyzer Type 2032 the measurement data is easily collected and analyzed. By applying the techniques of **Modal Analysis** to the set of measurements, a **modal model** can be derived, describing the dynamic properties of the structure.

Modal Analysis

Modal Analysis is the process of characterizing the dynamic properties of an elastic structure in terms of its modes of vibration. A mode of vibration is defined by its **modal frequency, modal damping** and **mode shape.** In theory, any deflection of the structure can be constructed as a linear combination of mode shapes. Loosely stated, the mode shapes represent the fundamental vibration patterns of the structure.

Modal analysis is an experimental method. Based on experience and expectations of the measurement results, a careful choice of measurement points is made. From the set of Frequency Response Function (FRF) measurements, the modal parameters are extracted, giving a simple mathematical model of the structure. The reduction of the measured data to modal parameters is done using matrix algebra and curve-fitting algorithms, a task most suitable for a computer. Eventually, the information from the measurements can be presented in a way which is easy to understand; for example, a single mode shape can be animated on the computer screen.

The following assumptions are made about the physical structure:

1. The structural motion can be adequately described by a set of ***linear*** second order differential equations.
2. The single modes can be separated. For each modal frequency there is only one associated mode shape.
3. The modal frequency and damping of each mode do not vary significantly across the structure.
4. Reciprocity between excitation and measurement points ensures that the transfer and system matrices are symmetric.

Using these assumptions, it is possible to describe the FRF matrix exclusively in terms of the modal parameters[1],[2]. It is not the intention to give a detailed explanation of the theory of modal analysis here. Many excellent references are available on the subject[1],[3]

With mechanical structures, the excitation is usually force applied with an impact hammer or vibration exciter (shaker). This is monitored with a force transducer. The response is usually acceleration, which can be measured with an accelerometer. Either excitation or response is fixed to a single location while moving the other from point to point to obtain the FRFs.

Measurement and analysis techniques

Before using modal analysis to examine the behaviour of a loudspeaker diaphragm, some practical problems had to be considered. The first was mass loading due to the transducer. Several methods have been investigated including ultra low-mass accelerometers and a non-contacting probe microphone. The effects of mass loading cannot be ignored unless the dynamic mass of the structure under investigation is several orders of magnitude greater than the transducer. For most drivers of polyethylene or paper construction, this mass is sufficiently low that even with the lightest accelerometer available (0,65g), mass-loading of the diaphragm still occurred. Although it is possible to correct for the effects of mass loading at a single point by using postprocessing, it is not possible to make the correction for a fixed excitation and a moving, mass loading transducer. Mounting such a transducer is also a problem, if the test is to be non-destructive. In the case of a loudspeaker, the excitation is intrinsically fixed (at the electrical input terminals, exciting the loudspeaker at the center of the diaphragm), so the ***transducer*** must move from point to point in order to collect the data. The effects of mass loading include a downward shift in the resonant frequency and changes in the damping.

The probe microphone technique also had limitations. Noise at the measured system output made it very difficult to obtain reliable data without extremely long averaging times. Another difficulty was the limited useful frequency range of the microphone itself. In performing such analyzes, it is assumed that the transducers used do not contribute to, or alter the response of the system under investigation. Use of the microphone also restricted the analysis to only "relative" measurements, meaning that the transducer output voltage could not be calibrated in terms of some physically measured parameter. The importance of this limitation will be shown.

These problems were alleviated by the use of a non-contacting laser velocity-transducer.

Test configuration

The test configuration is shown in Fig. 1. The driver was mounted in a baffle with the laser positioned approximately 50cm away. The measurements were made in the direction parallel to the main axis of motion (perpendicular to the baffle).

Response measurement

A Brüel & Kjær Dual Channel Signal Analyzer Type 2032 having 801 fre-

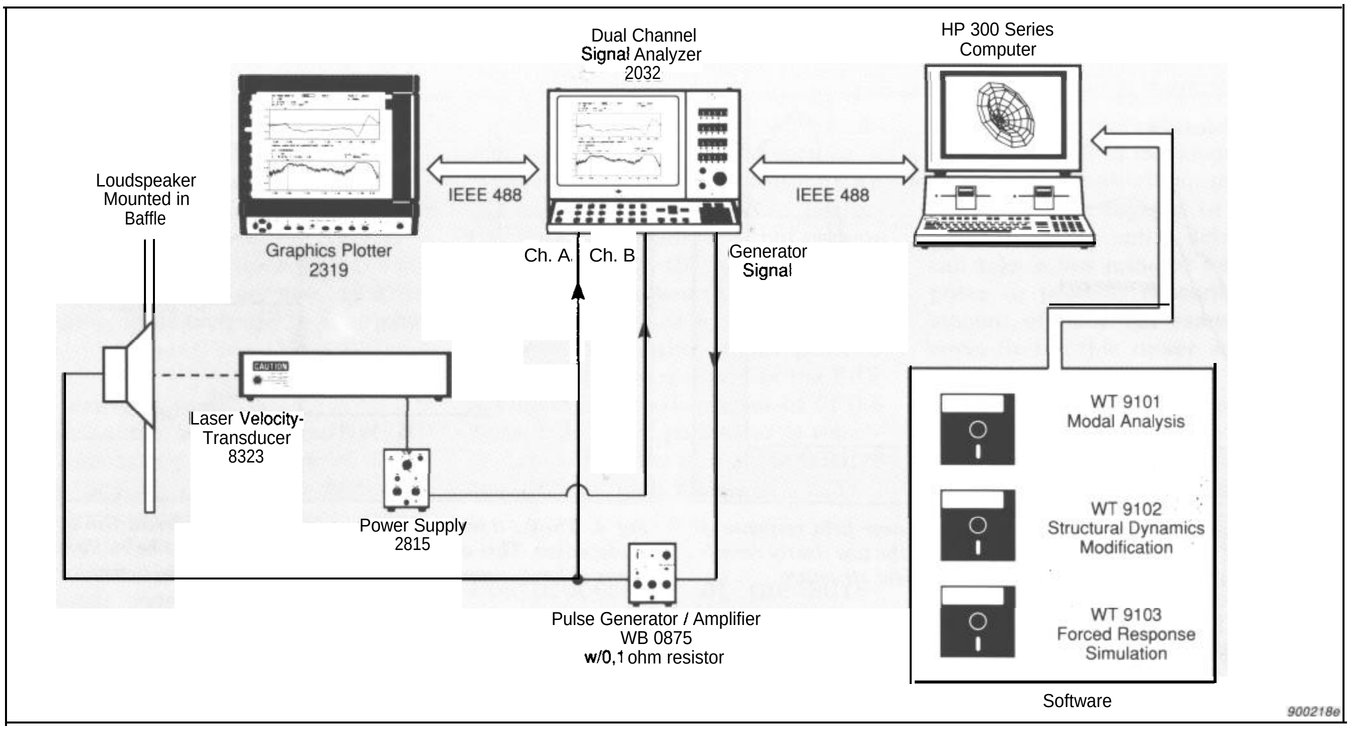

Fig. 1. Test configuration

quency lines was used for the analysis. The response was measured in channel B using a Brüel & Kjær Laser Velocity-Transducer Set Type 3544. The excitation was simultaneously measured in channel A. The analyzer can compute the frequency response from these measurements. There are several steps involved in performing a complete modal analysis, from establishing a geometry to producing an animation. A procedural diagram for a complete general modal test is shown in Fig. 2.

When measuring a vibrating object using the laser, a small piece of retro-reflective tape is mounted on the test object to return the source light from the target. The laser beam is split inside the device, one beam acting as the reference while the other is aimed at the target. The reference is Doppler shifted by a rotating disc and mixed with light returned from the target. A Doppler frequency shift also occurs in the target beam due to the vibration of the target. The two beams are sent to a photodetector and heterodyned. A detector converts this into a calibrated voltage proportional to the target velocity[4].

The response channel was calibrated by using a portable hand-held vibration calibrator, a Brüel & Kjær Calibration Exciter Type 4294. The output of the calibrator is a vibration signal of lOmm/s at a frequency of 159,2Hz. The output of the laser (nominally 1V/m/s) is fed to the analyzer, and the measured level is entered via the front panel as the calibration factor for channel B.

Excitation

The excitation used for the test was an electrical signal applied at the driver input terminals. Because loudspeaker drivers can exhibit non-linear behaviour, random noise was used to obtain the best approximation to the linear response of the system. For Modal Analysis as well as for FFT Analysis, the structure is assumed to behave in a linear manner or to be restricted to its linear range of operation. A noise excitation can also be band limited so that the system is only excited in the frequency range of interest, optimizing the dynamic range of the measurement instrumentation[5]. The driver chosen for this experiment was a Philips 10 inch woofer with a resonant frequency of 34,5 Hz.

For test purposes, the driver was mounted in a 1 m baffle. By comparing a measurement of the driver's near field response with a measurement made using an accelerometer mounted on the diaphragm (see Fig. 3), it is possible to find the frequency above which the driver no longer behaves like a rigid piston. The mass loading caused by the accelerometer is not critical in this case, as the measurement is for simple comparison purposes. The two measurements should have the same high-pass characteristic up to this frequency. Below this frequency, the near field response is proportional to acceleration[6]. This fre-

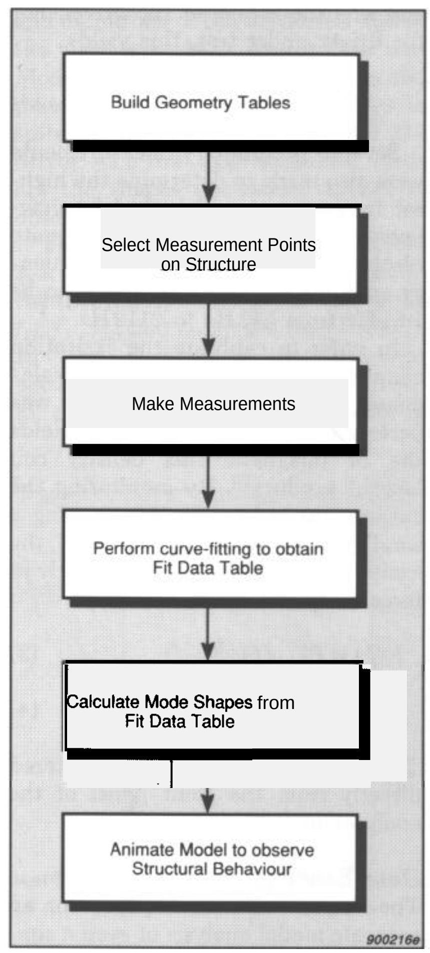

Fig. 2. Procedure for a general modal test

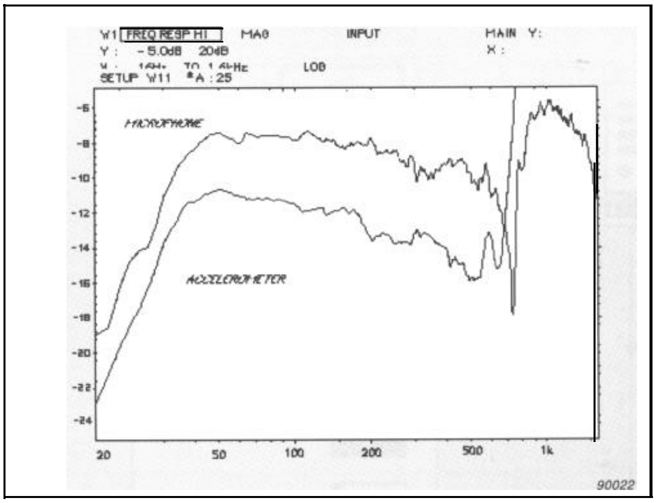

Fig. 3. *In the piston range of operation, the near field response is proportional to acceleration. A comparison of the two clearly reveals when the diaphragm no longer behaves as a rigid structure*

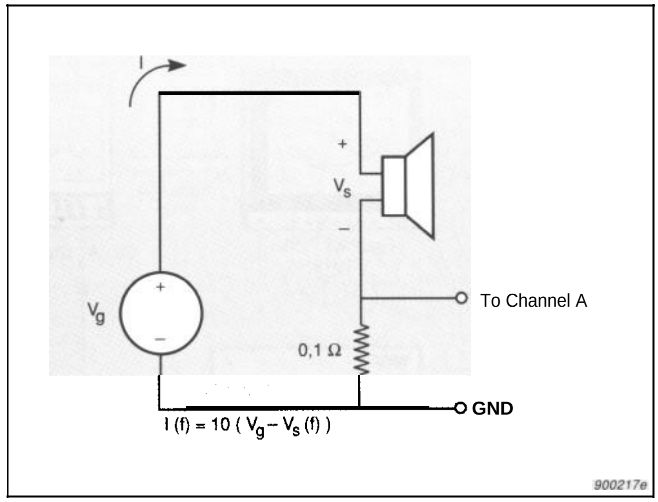

Fig. 4. *The 0,1 Ω resistor is used to measure the current through the loudspeaker. This allows the excitation channel to be calibrated in terms of force, since the force is proportional to the current*

quency could also be calculated by finding:

$$k \cdot a < 1 \qquad (1)$$

where k is the Wave Number $(2\pi/\lambda)$ and a is the radius of the driver. For the driver under test, this yields

$$f < 430\text{Hz} \qquad (2)$$

Several preliminary measurements were also made to determine the highest frequency at which modal behaviour could be observed with adequate resolution. Based on this, the frequency span for analysis was chosen to be 1,6kHz from 512Hz to 2112Hz.

In order to calibrate the excitation channel, a measurement of the Thiele - Small parameters of the driver was performed. This measurement yields the Bl (magnetic flux density * coil length) product [7]. By monitoring the current applied to the driver using a small (0,1 Ω) series resistor (Fig. 4), the excitation can be calibrated directly in force units (N) where

$$F(f) = Bl \,.\, i(f) \qquad (3)$$

$$= Bl \,.\, 0{,}1 \cdot v(f) \qquad (4)$$

This calibration factor can be entered directly from the front panel of the analyzer in V/N.

Data handling

The amount of data required for an accurate modal analysis of even a simple structure is large enough to justify the need for a computer. A total of 73 FRFs were measured at points every 30 degrees at six different radii on the driver (see Fig. 5). These FRFs are in the form of an analytic function. This means that the values for each FRF throughout the s-plane can be found from the known values along the frequency axis. The unknown modal parameters are identified by "curve-fitting" the measurements and synthesizing the transfer functions throughout the s-plane. This provides a mathematical definition of each transfer function based on the poles and residues determined for each of the resonances in the measured data [2]. From the curve-fit data, a "modal model", describing the response of the structure under investigation, can be developed.

By using a Hewlett Packard 300 Series computer and special application software (SMS MODAL 3.0 SE), a geometric "wire" model resembling the test object can be constructed using a convenient coordinate system. For our test object, the driver, a cylindrical coordinate system was chosen. The coordinates for our test were taken from a driver blueprint in order to represent the test object in a reasonable way. The geometry does not necessarily have to match the test object, but in most cases it does, at least approximately. The center point in our model represented the point of excitation or "driving point". The outermost diameter did not contain any measurement data and is used to represent the driver basket and provide a plane of reference for the motion.

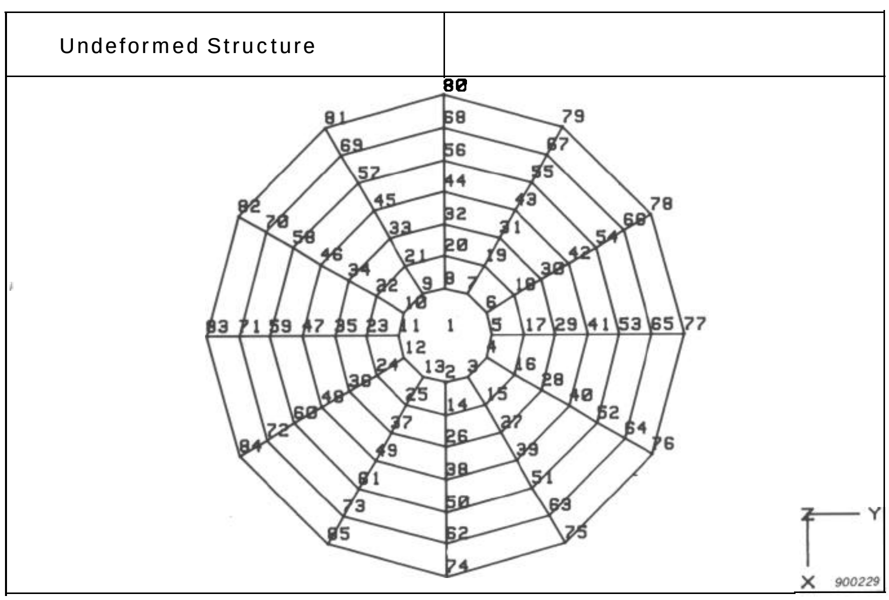

Fig. 5. *Measurements were performed at 73 different points around the diaphragm. No measurements were made at the points numbered from 74 to 85. These points represent the driver frame and are used as a reference for the displacement*

The software also facilitates data acquisition and storage. The software can then take the information from the modal model and use this to animate the geometry, giving a visual representation of the structure and its modal behaviour. Afterwards, the processed data can be stored on disk and used with other tools such as Structural Dynamic Modifications (SDM) and Forced Dynamic Response (FDR). The use of these tools will be described later.

The measurements
Measurements were performed at 73 points including a driving point measurement at the center of the dust-cap. This represented a reasonable compromise between required detail and measurement time. The dust-cap was oversized, extending approximately 1,5cm beyond the diameter of the voice coil. This unfortunately made it impossible to make any investigation on the diaphragm near the voice coil without destroying the unit. A small piece of retro-reflective tape ($\sim 0{,}25\,cm^2$) was moved to each position. This had no mass loading effects. Small positional adjustments of the laser were necessary at each position to improve the signal-to-noise ratio of each measurement.

Sharp, well-defined peaks appear in the magnitude response of most mechanical structures due to low damping. For these structures, identification of the modes is relatively simple. Besides the previously discussed problems of low mass (the total effective moving mass of the test unit was 34,3g), a loudspeaker driver is typically designed to have its modes heavily damped. In this case, location of the modes is somewhat more difficult.

A typical FRF is shown in Fig. 6. The real part of the frequency response and the measurement coherence are displayed. Coherence is a measure of how well the output of a system is linearly related to the input"]. As the measured output parameter was velocity, the real part of the response (as opposed to the imaginary or magnitude, which are also available) is chosen for viewing at this point in the analysis. The real part of the FRF is proportional to displacement (if the measured output parameter is velocity) providing some idea of the relative amplitude of each mode.

Postprocessing of measurement data

To a large degree, the quality and accuracy of the animated shapes obtained is dependent upon the curve-fit. Within the software, there are several possible types of curve-fitting routines. A choice must be made between Single Degree Of Freedom (SDOF) or Multiple Degree Of Freedom (MDOF) fitting, dependent upon the structure under test and amount of coupling between the modes. A choice is then made between Polynomial or Circle (on a Nyquist plot) fitting. For more complex or heavily coupled MDOF systems, a Complex Exponential and a Least Squares fit (operating on the Impulse Response) are available. By viewing several typical measurements or an average of a number of measurements, different ways of curve-fitting the data, can be attempted. The procedure is to position the cursors around one or several resonances and select a curve-fit routine. Once a reasonable fit has been obtained for all of the modes of interest in the sample data, an "Autofit" routine can be engaged to apply this curve-fit to the entire data set. This can take a few minutes for the computer to process, depending on the amount of data. An example of the curve-fit for this driver is shown in Fig. 7.

In the measured frequency range, which extended well beyond the intended usable frequency range for this unit, it was possible to identify seven modes above the rigid piston region. A list of the modal frequencies and damping appears in Table 1. Still views of the animated mode shapes, including the undeformed structure, are shown in Fig. 8 and Fig. 9.

The resulting mode shapes are viewed in terms of displacement, regardless of the units of the measured response. During the computer animation, it is possible to vary the speed and amplitude of the motion as well as the viewing angle. It can be seen that Modes l-3 correspond well to regular plate or stretched membrane modes, in spite of the cone construction of the driver. As a first approximation, these are the shapes one would expect to observe.

The modal frequencies for Modes 1,3,4, and 5 show good correlation to the theoretical frequency ratios for the f_{11}, f_{21}, f_{02}, and f_{03} modes (see Table 2). In this case, f_0 is approximately 480 Hz, just beyond the theoretical rigid piston limit [8]. The behaviour in

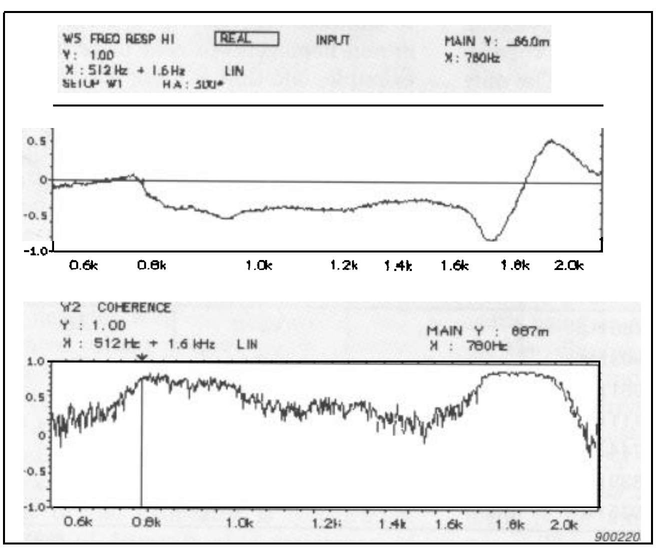

Fig. 6. A typical FRF measurement. For measurement point 24, the real part of the FRF and the coherence are shown

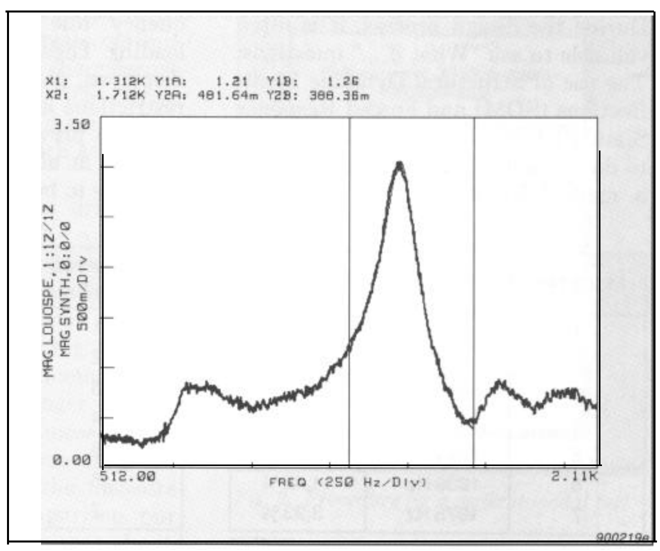

Fig. 7. An example of a curve-fit to a particular mode in a single FRF

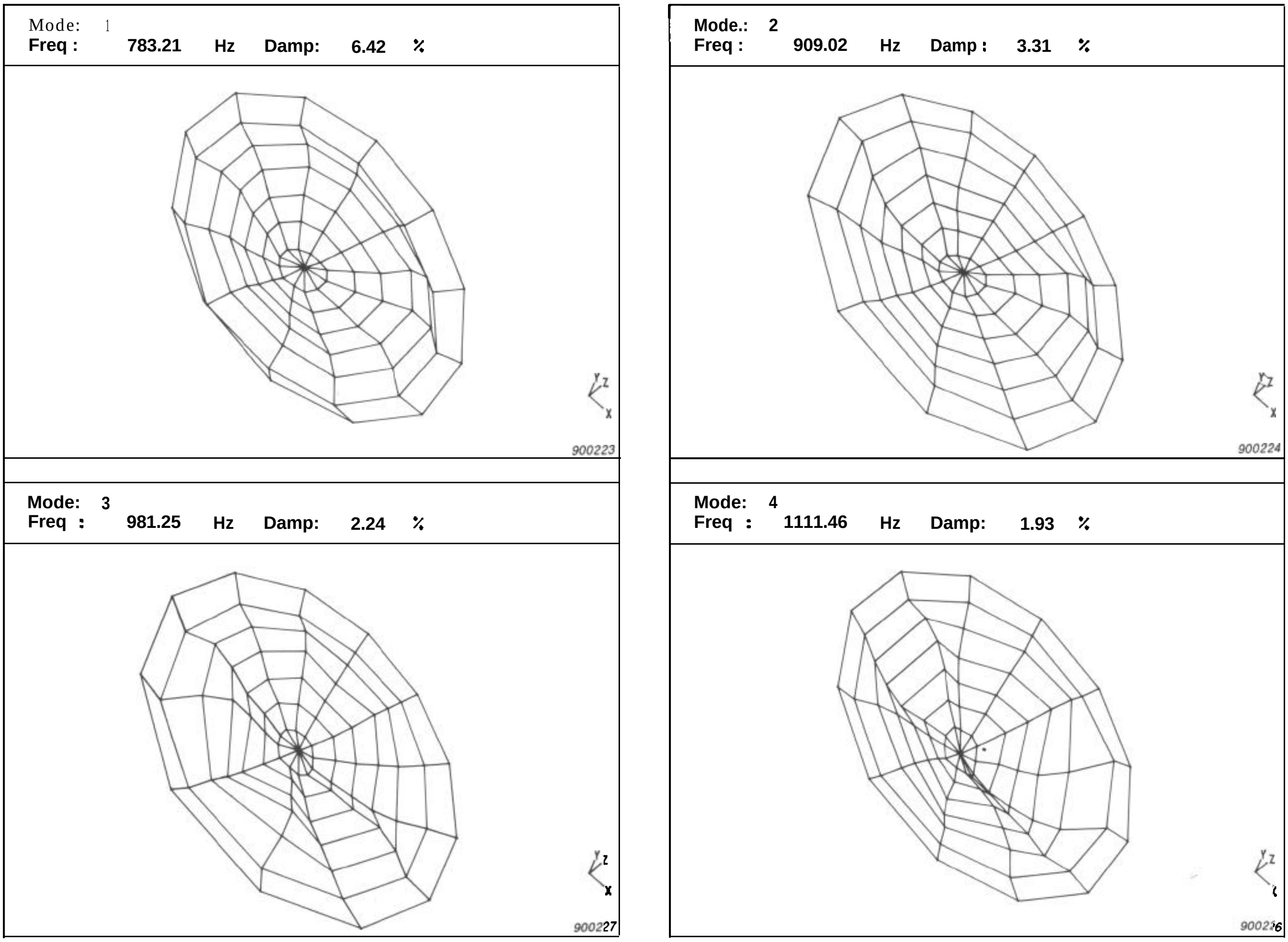

Fig. 8. Plots of the mode shapes 1, 2, 3, and 4. For each mode shape, the modal frequency and damping are listed above the structure

modes 4 and 6 is more irregular and is not immediately obvious. Plate behaviour is again seen in Modes 5 and 7, although the deformation near the dust cap in mode 7 is somewhat more severe than one would expect.

Implementation of SDM and FRS

During the design process, it is often valuable to ask "What if . . ." questions. The use of Structural Dynamic Modifications (SDM) and Forced Response Simulation (FRS) allows the engineer to do just that. For example, "What if a mass, stiffener or tuned absorber were added between two or more points on the structure? What would be the dynamic response of the modified structure?" It is then possible to predict analytically the structural response to real life excitations. In addition, it is possible to compute the resulting deformation at any single frequency due to static or dynamic loading. These simulations can then be displayed, plotted or stored. The only restrictions are that the mode shapes must be previously specified from an analysis at all points where modifications are to be made, and that the data used be obtained from calibrated inertial measurements of the force input and the corresponding acceleration, velocity, or displacement response[10]. This is why the calibration of the two channels is so important.

An example of a mass modification is shown in Fig. 10. This could be used to simulate the effect of a progressive or non-homogeneous cone material, for example, one that is more massive toward the voice coil than at the surround. The applied mass may be either positive or negative, at one or several points. A negative mass might be

Mode No.	Freq.	Damping
1	703 Hz	6,42 %
2	909 Hz	3.31 %
3	981 Hz	2,24 %
4	1111 Hz	1,93 %
5	1744Hz	3,19%
6	1839 Hz	4,42 %
7	1975 Hz	3.23 %

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Tablel. Modal frequencies and dampzngs for the first 7 modes.

Measured Mode	Measured Freq.	Theoretical Mode	Theoretical Freq.	Ratio f/f_0
1	783 Hz	f_{11}	763 Hz	1,59
2	909 Hz			
3	981 Hz	f_{21}	1027 Hz	2.14
4	1111 Hz	f_{02}	1104Hz	2,30
5	1744Hz	f_{03}	1728 Hz	3,60
6	1839 Hz	–	–	–
7	1975 Hz	–	–	

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Table2. Comparison between the measured modal frequencies and the theoretical eigenfrequencies for a stretched membrane.

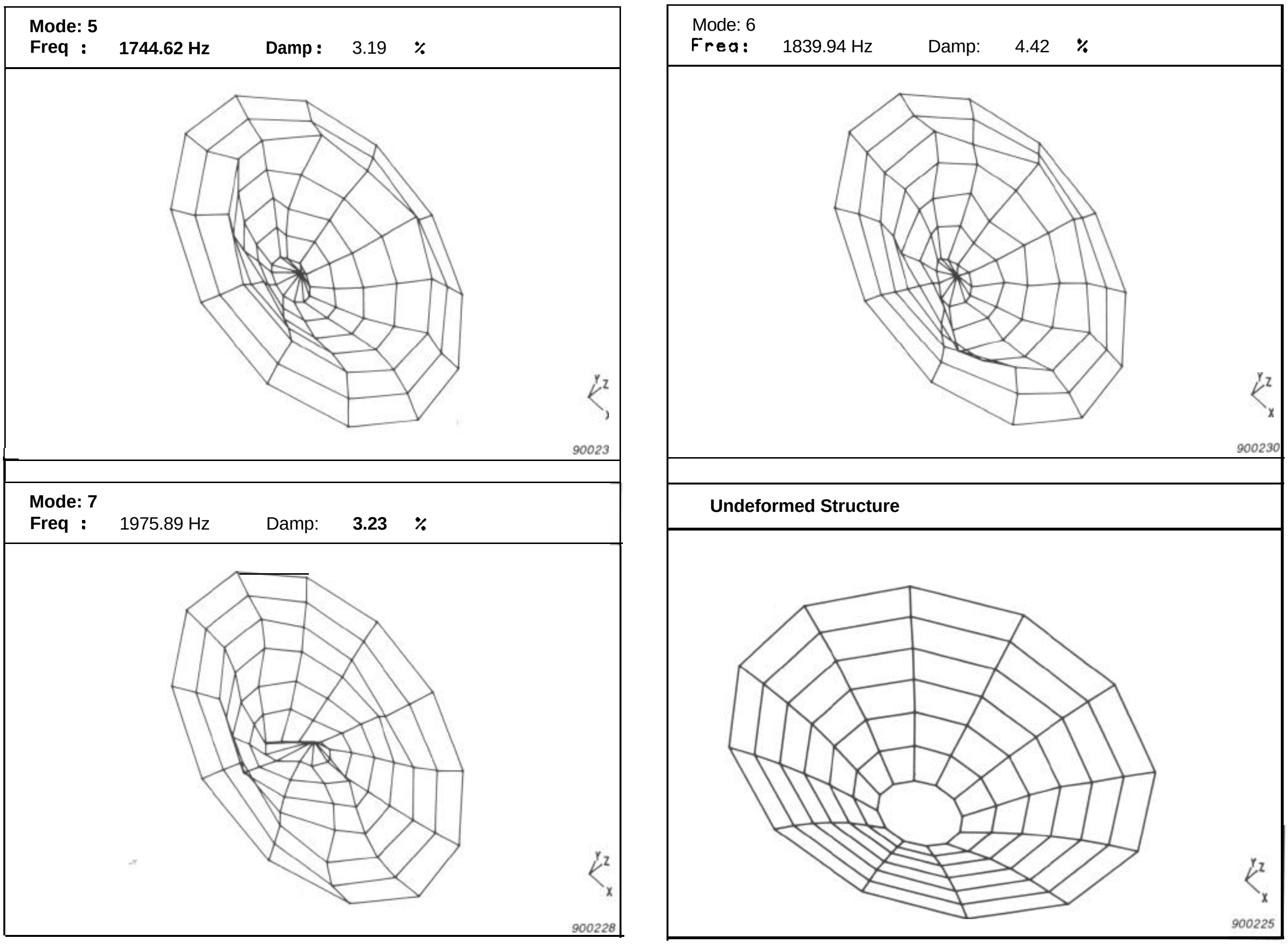

Fig. 9. Plots of the mode shapes 5, 6, 7 and the undeformed diaphragm.

used to compensate for a manufacturing defect where too much cone material has gathered at some point. Several different modifications could be combined.

The use of a stiffness modification could be used, for example, to simulate the effects of a different surround material. This is shown in Fig. 11. After the application of the desired modifications, the response of the modal model at any frequency can be simulated (with or without the modifications) using FRS.

The presence of modal behaviour has a definite effect on the driver's acoustic performance. In general, the primary concern in driver design is to extend the region of rigid piston operation as high as possible with a minimum of compromise to other design factors. To maximize the overall usable frequency range of the driver, it is important that beyond the piston mode, the next few modes do not cause the acoustic efficiency to significantly deteriorate. Excessive amounts of

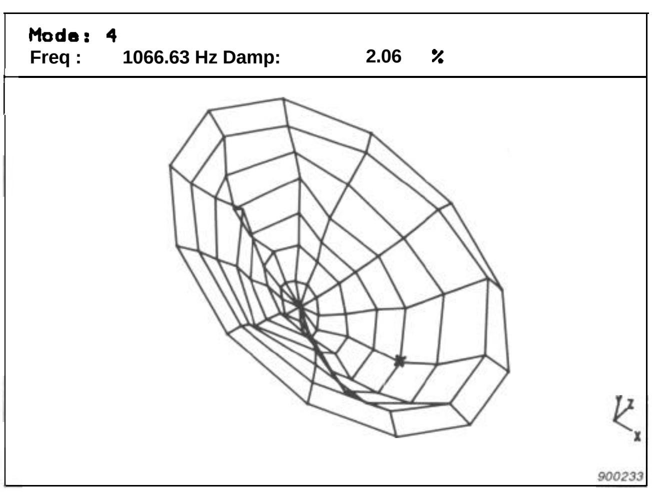

Fig. 10. Example of an added mass simulation (50g at point 38). Note the downward shift in modal frequency

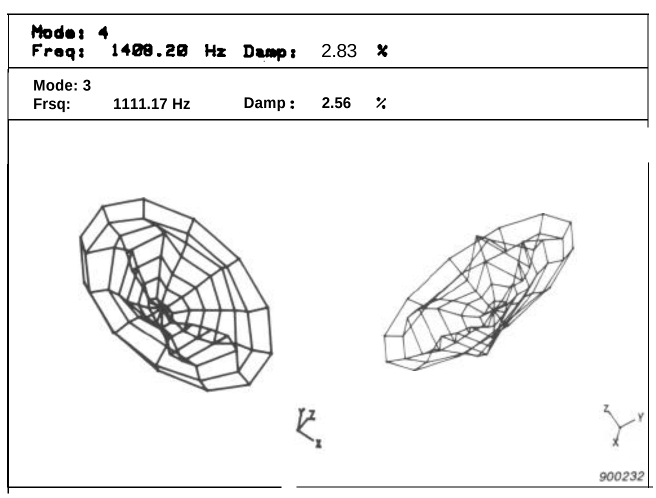

Fig. 11. Example of a stiffness modification near the surround to simulate a change in material. Note the upward shift in modal frequencies

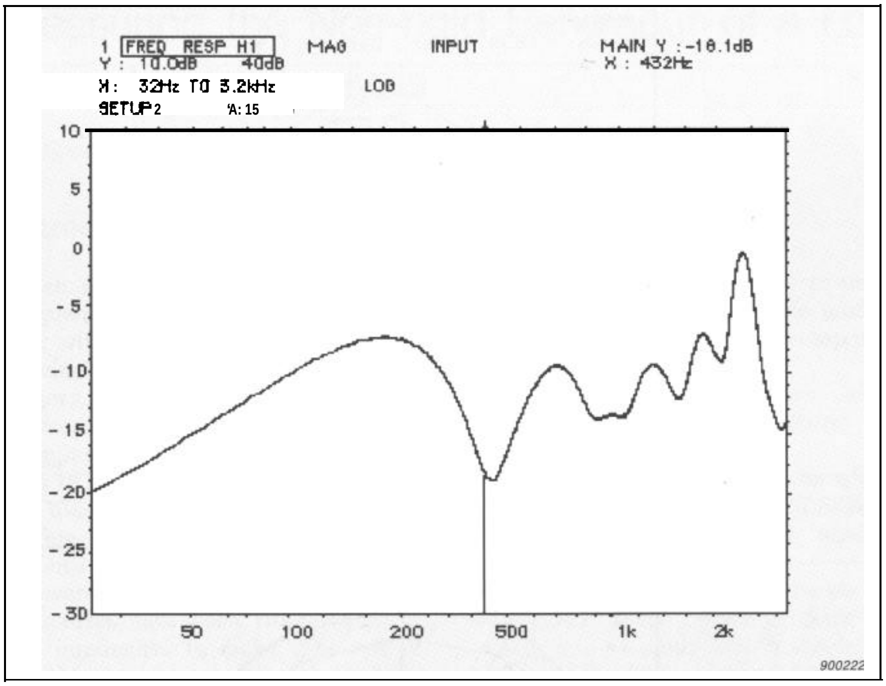

Fig. 12. A measurement showing the free field response of the driver. Structural modes contribute to the uariations in the response beyond the rigid piston range of operation

counterphase motion, where large portions of the diaphragm are out of phase with the excitation, will produce acoustic cancellations in the near field. This in turn results in dips in the free field frequency response of the driver (see Fig. 12) and an overall lowering of its efficiency.

Applications

It is possible to perform such a modal analysis on a known, well behaved driver and use this as a reference. By performing the analysis on similar units or perhaps slightly modified prototypes for new models, defects in construction or manufacture can quickly be identified. In this way, improvements in design can also be studied in detail.

If a detailed analysis of a particular problem is required, an exact geometric model of the device under investigation can be entered. The analysis can then be restricted to the specific location and frequency range of interest.

Modal analysis can also be used to verify the accuracy of analytical models constructed using Finite Element Methods. An example of such a model and its calculation is given by Kaizer and Leeuwestein[9].

Conclusions

Modal analysis is a powerful tool for the analysis of the dynamic behaviour of structures. Although normally applied to the study of relatively massive and lightly damped mechanical structures, it can easily accommodate the analysis of a highly damped, low-mass device such as a loudspeaker driver. This is made possible by the use of a non-contacting laser velocity-transducer.

In this Application Note, a complete step by step analysis for a typical loudspeaker driver was carried out. This unit was shown to exhibit modal behaviour similar to the familiar plate modes. The use of the laser transducer allows calibrated response measurements to be performed. A systematic method for calibrating the input excitation in terms of force has also been shown. The calibration of the measurements is clearly important, as it makes it possible to apply structural dynamic modifications and observe the forced dynamic response of the system. These modifications can be used to simulate and solve actual design problems.

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